A Miniaturized X-band High-Index Supercavity Resonator in Microstrip Technology

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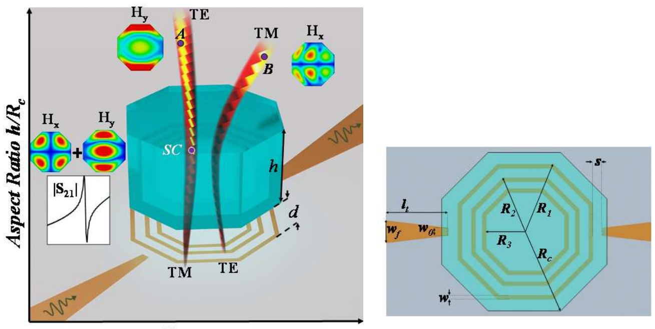
***Abstract*—We present the first supercavity-mode high-index resonators excited by microstrip structures at X-band. Supercavity modes are attractive for trapping energy in high-quality subwavelength resonators. Previously this localization option was only demonstrated in three-dimensional resonators in the RF regime. Enabling such modes in planar technologies requires significant miniaturization and novel coupling structures that can excite the required nonorthogonal mode pairs. This work addresses both of these challenges for the first time. Specifically, we employ microstrip resonators to excite hybrid supercavity modes. We also demonstrate the robustness of the presented design with respect to fabrication uncertainties. Furthermore, we employ microstrip high-index resonators (HIR) with dielectric constant of 140. A proof-of-concept resonator with volume of less than 3.45 × 10^{-4} λ^3 and first supercavity resonance of 11.2 GHz with a measured unloaded quality factor of 204 is demonstrated. This is the highest reported quality factor in planar-excited supercavity technology with the above-mentioned volume.**

Keywords— Supercavity modes, X-band resonators, high index dielectric resonators.

I. INTRODUCTION

Subwavelength high-quality resonators are of critical practical importance in applications requiring miniaturized antennas, matching networks, and filters [1]. Microstrip resonators are commonly used due to their small size and low fabrication cost. However, their quality factors are fundamentally limited by their metal and dielectric losses [2]. Dielectric resonators (DRs) are an attractive alternative since they do not suffer from metal losses [2]. Nevertheless, their quality factors are still limited in the microwave range because of dielectric and radiation losses.

High-index resonators (HIRs) are miniaturized structures that enable localizing electromagnetic modes originating from so-called electric and magnetic Mie resonances. Radiation loss in HIRs can be suppressed via destructive interference between two or more radiating modes with similar radiation profiles. This mechanism is well described within the framework of the bound states in the continuum (BICs) theory. Exotic nonradiative states with extraordinarily high-quality factors can be possible [3]. However, the finite dimensions of real-life resonators give no possibility to implement such dark modes (pure BICs). For a single HIR, an analogy of BICs in the form of supercavity (SC) modes (quasi-BIC) can be excited with unusually-high quality factors (> 1,000) [3]–[5].



(a) (b)

Fig. 1. (a) Proposed microstrip-HIR and observed mode transformation at quasi-BIC. (b) Yop view of the schematic of the proposed resonator. The concentric octagonal resonators are printed on a 25-mil TMM6 substrate with $R_1 = 1.9$ mm, $R_2 = 1.4$ mm, $R_3 = 1.0$ mm, $w = 120$ µm, $s = 0.1$ mm, $h = c \times Rc$ and $w_0 = 0.2$ mm, $w_f = 2.1$ mm, $l_t = 8$ mm.

While three-dimensional SC mode resonators have been successfully demonstrated in radio frequencies [5], no such demonstrations have appeared in microstrip-HIR technology. In this work, we present the first such successful demonstration at X-band. The key to our approach is the successful excitation of appropriate nonorthogonal modes that support quasi-BICs in significantly miniaturized subwavelength HIRs. To validate our approach, a microstrip-HIR operating at X-band is designed and fabricated for the first time with a measured quality factor of 204 and volume of 3.45 × 10^{-4} λ^3.

II. MICROSTRIP-HIR RESONATOR STRUCTURE

Fig. 1 depicts the concept of the proposed quasi-BIC-based microstrip-HIR design. The HIR is excited by three strongly-coupled microstrip ring resonators. Employing multiple strongly-coupled rings is key for exciting the needed HIR modes. This microstrip source can excite an appropriate combination of coupled high-order axisymmetric modes with a resulting quality factor more than 2× higher than a pair of conventional simple low-order modes in dielectric resonators [2]. Moreover, the optimum microstrip-ring-HIR coupling is achieved by co-designing the HIR and microstrip rings to generate nonorthogonal pairs of modes (including TE and TM polarizations) that can cancel each other in the far field. Having such rich collection of modes increases the probability of exciting appropriate supercavity modes for miniaturized HIRs.

Although destructive mode interference with complementary radiation profiles requires careful design, the proposed method is not restricted to the specific geometrical shape of HIR or excitation source; any geometry with rotational symmetry, such as circular, octagonal, and square rings/cylinders, can be chosen, but with some penalty in the achieved quality factor. The proposed design employs an octagonal shape as a trade-off between fabrication complexity and quality factor.

As discussed in [6], inducing destructive interference between pairs of modes with overlapping radiation profiles causes these modes to hybridize producing two new modes with so-called avoided mode crossing (AMC) in their energy dispersion spectra. This phenomenon is shown in Fig. 1 (a), where a TM mode smoothly transforms to TE mode through a quasi-BIC (SC mode) with a hybrid field distribution. In the vicinity of this singularity point, the damping rate for one of these modes becomes zero implying a theoretically infinite quality factor. This is the underlying mechanism for BIC. Similarly, the supercavity regime (quasi-BIC) is observed close to AMC of the wave's spectrum, where two resonant modes are coupled to the same radiation channel [7], [8]. This interference mechanism and its spectral features can be mathematically described using a physically intuitive picture of coupled, damped harmonic oscillators. The Hamiltonian of such a system can be written as:

$$H = \begin{pmatrix} \omega_1 & \kappa \\ \kappa & \omega_2 \end{pmatrix} - \mathrm{i}\begin{pmatrix} \gamma_1 & \sqrt{\gamma_1\gamma_2}\mathrm{e}^{\mathrm{i}}\psi \\ \sqrt{\gamma_1\gamma_2}\mathrm{e}^{\mathrm{i}}\psi & \gamma_2 \end{pmatrix} \tag{1}$$

Here, ω_i and γ_i are the resonant frequencies and the damping rates (∝ 1/Q) of the modes $i = 1, 2$, respectively. ψ is the phase shift between the modes and κ is the near-field coupling factor between two modes. $\sqrt{\gamma_1\gamma_2}e^{i\psi}$ is defined as the far-field coupling between two modes. From the determinant condition for the system's Hamiltonian, the avoided band is obtained when $\omega_1 = \omega_2$ and $\gamma_1 = \gamma_2$ where the two modes will approach the same frequency but are not identical. This condition provides non-zero off-diagonal elements in the Hamiltonian formulated in (1). Hence, to observe the AMC, the HIR dimensions should be engineered to support pairs of modes with the $\omega_1 = \omega_2$ condition. This criterion constrains the resonators' geometrical parameters and determines limits on its possible miniaturization. For instance, Fig. 2 shows the dependence of the resonant frequency of different modes in the axially symmetric cylindrical rod resonator with radius R_c and thickness h in terms of the aspect ratio (AR) defined as $AR = {}^{h}/_{R_c}$. The TM_{121} and TE_{021} modes are the first two nonorthogonal modes with nearly identical frequencies in a HIR made from rutile TiO_2 (ε_r~140 and $tan\delta$ ~ 6×10^{-4} [9]) with radius R_c < 10% λ (wavelength at the mid-X band). Their frequencies remain almost identical for a wide range of resonator aspect ratios. This underlines the robustness of the proposed structured to fabrication uncertainties. For lower ε_r ($\varepsilon_r < 100$), the first X-band nonorthogonal modes appear for R_c> 15% λ. Thus, high-ε_r dielectrics should be considered for optimal miniaturization.

Once the desired modes have been identified, the proposed microstrip-HIR structure is engineered to hybridize the TM_{121} and TE_{021} modes by optimizing the rings-HIR coupling. This is accomplished by controlling the AR of the HIR. Naturally, an AR that minimizes the cavity volume is desirable. To optimize the design we model the structure in the CST Microwave Studio®. Figs. 3(a)-(d) shows the simulated $|S_{21}|$ maps for different ARs and clearly indicate the SC region. The first SCs appear around the intersection of modes close to the first AMC.

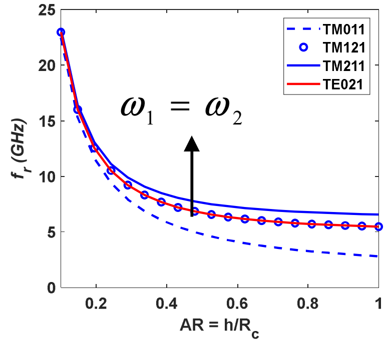
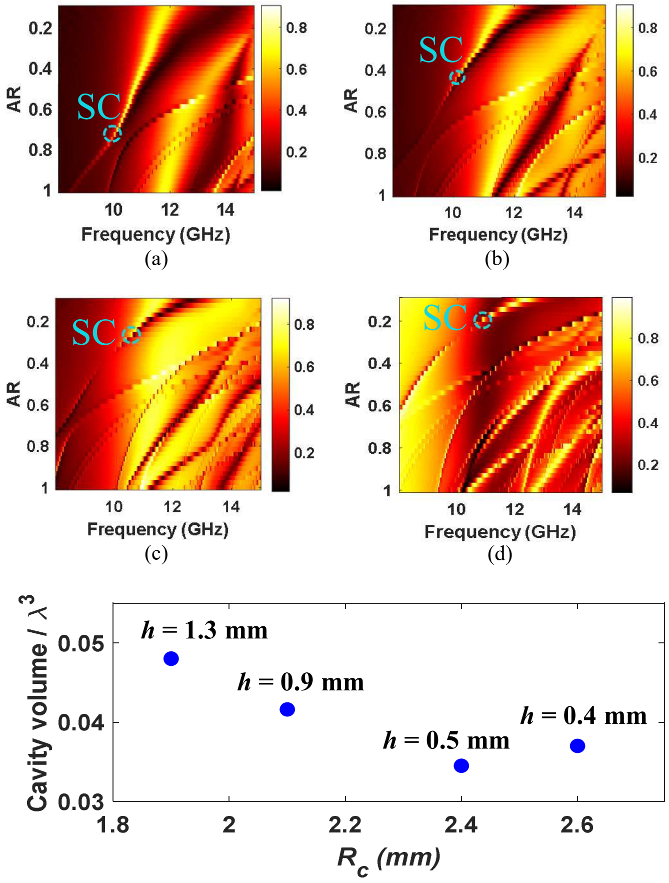


Fig. 2. Calculated the resonant frequency of different modes in the HIR at the system with 0th-order rotational symmetry.



(a) (b) (c) (d) (e)

Fig. 3. Simulated $||S_{21}|$ map in linear scale for three configurations with (a) $R_c = 1.9\,mm$, (b) $R_c = 2.1\,mm$, (c) $R_c = 2.4\,mm$, (d) $R_c = 2.6\,mm$. (e) calculated cavities volumes / λ^3 at SC is shown as a function of R_c.

Fig. 3(e) summarizes the simulated results by showing the HIR normalized volume for the first SC as a function of its radius R_c. As seen, the optimum R_c is around 2.4 mm. The corresponding resonator heights are also shown in Fig. 3(e).

Next, simulations are performed by tuning the separation distance between the HIR and microstrip excitation (*d*), hence adjusting the SC resonant frequency (Fig. 4). Notice that these results refer to weakly coupled resonators to identify resonant frequency and quality factor. The observed sensitivity of the SC regime to *d* enables tuning of the structure to precisely achieve the desired resonant frequency without changing the resonator's thickness. The distance *d*, however, also mildly affects the resonator's quality factor. The optimum d to have maximum quality factor (~ 240) is 120 µm in this design.

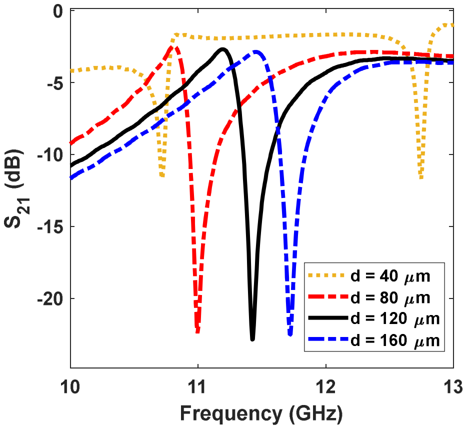


Fig. 4. Simulated $|S_{21}|$ of weakly coupled resonator with $c \sim 0.2$ $and\ R_c = 2.4\ mm$ for different d values.

III. FABRICATION AND EXPERIMENTS

The proposed design has been fabricated to experimentally validate the theoretical concept. The high-index crystal (TiO_2 cavity) is mounted on a dicing tape and cut using a resin blade at 2 mm/sec feed speed. To conduct the measurement, the TiO_2 crystal is held with a dielectric vacuum tip, aligned, and placed on top of the resonator using pick-n-place equipment by FineTech. The distance of the crystal from the microstrip resonator is tuned by using a z-micromanipulator of the stage. Initially, the crystal is zeroed on the substrate, and then, the gap is increased (Fig. 5). Measurements are performed by tuning *d*, hence adjusting the SC resonant frequency. As discussed before, the highest unloaded quality factor (~240) is achieved when *d* ~ 120 μm. A measured quality factor of about 208 is obtained. This is approximately 2× than the calculated Q-factor of the suggested design in [2] at X-band. The differences between simulation and measurement results come primarily from imperfections in dicing the octagonal shape of the TiO_2 crystal, alignment errors, and the dielectric vacuum tip effect.

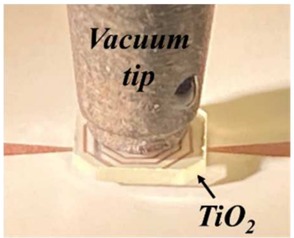


Fig. 5. Setup for measuring resonances in an X-band supercavity filter.

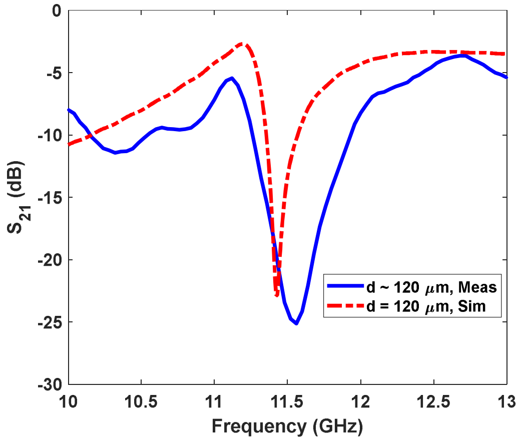


Fig. 6. Measured and Simulated $|S_{21}|$ of weakly coupled resonator with $c \sim 0.2$ $and\ R_c = 2.4\ mm.$

IV. CONCLUSION

In this work, we presented the first supercavity high-index resonators excited by microstrip structures at X-band. The microstrip-HIR relies on the appropriate nonorthogonal modes (TM_{121} and TE_{021}) that have nearly identical frequencies for a wide range of aspect ratios and can yield the desirable first supercavity resonance at 11.2 GHz within a miniaturized structure with a volume of less than $3.45 \times 10^{-4}\ \lambda^3$. A measured quality factor of 204 (85% of the theoretical maximum) is achieved. The presented concept can open new directions in miniaturized supercavity resonators in planar technologies.

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